FREE FALL RATE OF A SPHERICAL PARTICLE IN AN IMMOBILE LIQUID

A. D. Al'tshul' and Yu. A. Voitinskaya

UDC 532.55

A derivation is offered for a semiempirical formula for the drag coefficient of a sphere which is effective over the entire precrisis range, and has not previously been theoretically justified.

The formula for determination of the free-fall velocity W of a solid particle of diameter d in an immobile liquid has the form [2]

$$\frac{W}{\sqrt{gd}} = \sqrt{\frac{4}{3}}\sqrt{\frac{\rho_s - \rho_l}{\rho_l C_d}}, \tag{1}$$

where g is the acceleration due to gravity; ρ_s is the density of the solid particle; ρ_l is the density of the liquid; and C_d is the particle drag coefficient, for which a large number of empirical formulas have been proposed.

The concept of the so-called generalized Reynolds number, introduced in 1958 [1], permitted solution of a number of hydraulic problems, including that of determination of the drag coefficient of objects upon their fall through a liquid. Over the entire precrisis range a semiempirical formula was obtained for the drag coefficient of a sphere [2, 3]:

$$C_d = \frac{24}{\mathrm{Re}} + 0.67\sqrt{C_d}, \tag{2a}$$

where Re = Wd/ν; ν is the kinematic viscosity of the liquid.

Equation (2a) is simply transformed to the form

$$C_d = 0.112\left(1 + \sqrt{1 + \frac{214}{\mathrm{Re}}}\right)^2. \tag{2b}$$

Substituting Eq. (2b) in the expression for fall rate (1), after transformations we obtain

$$W = \frac{ga^2d^2}{18\nu + 0.6a\sqrt{gd^3}}, \tag{3}$$

where

$$a^2 = \frac{\rho_s}{\rho_l} - 1. \tag{4}$$

Introducing the notation

$$\mathrm{Ar} = \frac{gd^3}{\nu^2}a^2, \tag{5}$$

where Ar is the Archimedes number, instead of Eq. (3) we have

$$W = \frac{\nu}{d}\frac{\mathrm{Ar}}{18 + 0.6\sqrt{\mathrm{Ar}}}. \tag{6}$$

A formula similar to Eq. (6) was obtained previously [4] with the aid of mechanical interpolation between formulas for purely viscous and inertial flow around spheres, and has not been justified theoretically until the present. Special investigations have confirmed that both Eq. (6), and Eq. (2a), from which the former is derived, agree very well with experimental data over the entire precrisis range of drag [3, 4].

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 35, No. 4, pp. 741-742, October, 1978. Original article submitted October 11, 1976.

 0022-0841/78/3504-1258$07.50

LITERATURE CITED

1. A. D. Al'tshul', Inzh.-Fiz. Zh., 1, No. 2 (1958).
2. A. D. Al'tshul' and P. G. Kiselev, Hydraulics and Aerodynamics [in Russian], Stroiizdat (1965).
3. A. D. Al'tshul', in: Collected Works of the Kuibyshev MISI No. 55 "Motion of Homogeneous and Inhomogeneous Liquids," Vol. 1 [in Russian], Moscow (1968).
4. R. B. Rozenbaum and O. M. Todes, Zap. Leningr. Gern. Inst., 36, No. 3 (1958).

NONSTATIONARY NONLINEAR HEAT-CONDUCTION PROBLEMS

A. A. Puchkov

UDC 517.9:536.21

An exact analytical solution is constructed for the one-dimensional nonlinear nonstationary problem of heat conductivity with boundary conditions of the third kind.

The solution of the nonstationary one-dimensional problem of heat conductivity in the absence of heat sources and sinks within the body results in the need to investigate the equation [1]

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial \mu}{\partial r} \cdot \frac{\partial T}{\partial r} + \frac{\nu \mu}{r} \cdot \frac{\partial T}{\partial r} + \mu \frac{\partial^2 T}{\partial r^2}. \tag{1}$$

Let us find the temperature T = T(r, t) satisfying this equation, the initial condition

$$T(r, 0) = \varphi(r) \tag{2}$$

and the boundary conditions of the third kind

$$\left(\mu \frac{\partial T}{\partial r} + \mu_1 T\right)_{r=l} = 0; \quad \left(\mu \frac{\partial T}{\partial r} - \mu_2 T\right)_{r=-l} = 0, \tag{3}$$

where the heat-exchange coefficients $\mu_1 = \mu_1(r, t, T)$ and $\mu_2 = \mu_2(r, t, T)$ are given or empirical parameters.

Limiting ourselves to an examination of the symmetric problem, let us take [1]

$$\left(\frac{\partial T}{\partial r}\right)_{r=0} = 0. \tag{4}$$

in place of the second condition in (3). The values $\nu = 0, 1, 2$ determine the plane, cylindrical, or spherical symmetry of the body, respectively.

Let there be the dependences

$$\rho c = P(t) F(r) T^p; \quad \mu = Q(t) G(r) T^q. \tag{5}$$

Then (1) is reduced to the form

$$\frac{T^{p-q+1}}{\lambda(t)} \cdot \frac{\partial T}{\partial t} = \frac{G(r)}{F(r)} \left\{T \frac{\partial^2 T}{\partial r^2} + \left[\frac{G'(r)}{F(r)} + \frac{\nu}{r}\right] T \frac{\partial T}{\partial r} + q \left(\frac{\partial T}{\partial r}\right)^2\right\}, \tag{6}$$

where $\lambda(t) = Q(t)/P(t)$.

We construct the solution of (6) in the form of the product

$$T = \tau(t) R(r). \tag{7}$$

Inserting (7) into (6) and separating variables, we obtain two equalities

$$\tau^{p-q-1} \frac{d\tau}{dt} = -k^2 \lambda(t), \tag{8}$$

Kiev Polytechnic Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 35, No. 4, pp. 743-746, October, 1978. Original article submitted June 2, 1977.